

## UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 7TH FEBRUARY 2002

Organised by the

**United Kingdom Mathematics Trust**  
from the School of Mathematics, University of Leeds



### **RULES AND GUIDELINES** (to be read before starting)

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **1 hour**.  
No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 11 or below.  
Candidates in Scotland must be in S4 or below.  
Candidates in Northern Ireland must be in School Year 12 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.  
Six marks are awarded for each correct answer to Questions 16-25.  
**Each incorrect answer to Questions 16-20 loses 1 mark.**  
**Each incorrect answer to Questions 21-25 loses 2 marks.**
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. The UK IMC is about solving interesting problems, not about lucky guessing.

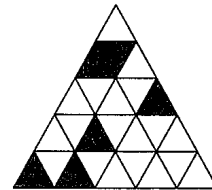
**The UKMT is a registered charity**

1. Which of the following numbers could replace  $\square$  so that the value of  $\frac{\square}{5}$  lies between 3 and 4?  
 A 3.2            B 9            C 14            D 19            E 24

2. Which of the following has the greatest value?  
 A  $0.3 \times 7$       B  $0.5 \times 5$       C  $0.2 \times 11$       D  $0.09 \times 30$       E  $0.026 \times 100$

3. Alice's room is furnished with three-legged stools and four-legged chairs. There are 17 legs in all (excluding Alice's!). How many three-legged stools are there?  
 A 1            B 2            C 3            D 4            E 5

4. The figure shows an equilateral triangle divided into small equilateral triangles, all equal. What is the lowest number of small triangles which must now be shaded to produce a figure which has a line of symmetry?



- A 2      B 3      C 4      D 5      E 6

5. My local greengrocery, 'Apples and Pears' (known as 'AP' for short), charges 24p for the first apple bought, 23p for the second, 22p for the third, and so on, each apple costing 1p less than the one before. How much change should I receive if I buy 9 apples and I give the shopkeeper £2?

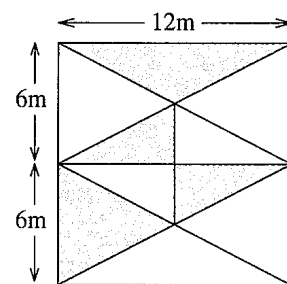
- A 24p            B 23p            C 22p            D 21p            E 20p

6. The *letter-product* of a number is obtained by multiplying the number by the number of letters in the corresponding word. For example, the letter-product of 5 is 20, since there are 4 letters in the word 'five' and  $5 \times 4 = 20$ . Which of the following has the largest letter-product?

- A 6            B 7            C 8            D 9            E 10

7. In square metres, what area of this pennant is shaded grey?

- A 50      B 54      C 57      D 60      E 72



8. What is the value of  $2^{10} - 10^2$ ?

- A -1000            B -80            C 0            D 924            E 1000

9. A *Langford number* is one in which each digit of the number occurs twice; the digits 1 are separated by one other digit, the digits 2 are separated by two others, and so on. Which of the following is a Langford number?

- A 12142334      B 41312432      C 14132342      D 32432141      E 31213244

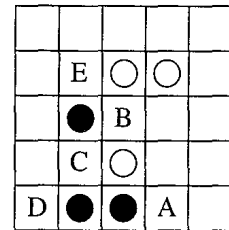
10. Anna has 3 brothers and 5 sisters. Her brother Tom has  $S$  sisters and  $B$  brothers. What is the value of  $S \times B$ ?

- A 8            B 10            C 12            D 15            E 18

11. The standard Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, ... begins with two 1s, and each later number in the sequence is the sum of the previous two numbers. Other Fibonacci-like sequences can be constructed by starting with any two numbers  $a$  and  $b$  (not necessarily 1 and 1) and using the same rule for creating the other numbers in the sequence. What is the first term of the Fibonacci-like sequence whose second term is 4 and whose fifth term is 22?

A 2                      B 3                      C 4                      D 5                      E 6

12. In the game Four-in-a-Row, two players take it in turns to place counters on the  $5 \times 5$  board. The winner is the first player to have four adjacent counters in a line across or down (but not diagonally).



It is Black's turn to play next. Where should she play her fourth counter to be certain of winning on her fifth turn whatever White plays?

A                      B                      C                      D                      E

13. Granny has made another of her special super-heavy giant rock cakes. At her birthday party, five of the guests tried to guess the weight of the cake. Their guesses were 5040g, 5060g, 5110g, 5120g, and 5150g. Actually, none of them was right. Only two were more than 30 grams out, and they were out by 70g and 90g. What was the weight of the cake?

A 5070g                      B 5080g                      C 5090g                      D 5110g                      E 5130g

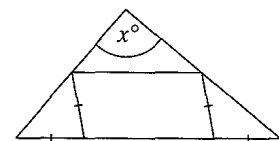
14. I have four rectangular pieces of thin hardboard whose dimensions (in cm) are  $55 \times 85$ ,  $65 \times 75$ ,  $65 \times 85$  and  $90 \times 105$ . Without bending the hardboard, how many of these can I get through an open rectangular window measuring  $60 \text{ cm} \times 80 \text{ cm}$ ?

A 0                      B 1                      C 2                      D 3                      E 4

15. Jack had five cards in a pile on a table. He gave me the top card, and then placed the next card at the bottom of the pile; then he gave me the next one on the top and placed the next one after that at the bottom of his pile. He continued like this until he had given me all of the cards. I looked down and to my surprise found that Jack had given me the cards in order: Ace, 2, 3, 4, 5. In what order (top to bottom) did Jack originally have the cards in the pile on the table?

A Ace,2,3,4,5                      B Ace,4,2,5,3                      C Ace,5,2,3,4                      D Ace,5,2,4,3                      E Ace,5,3,4,2

16. The diagram, which is not drawn accurately, shows a parallelogram inside a triangle. The marked lengths are equal. What is the value of  $x$ ?



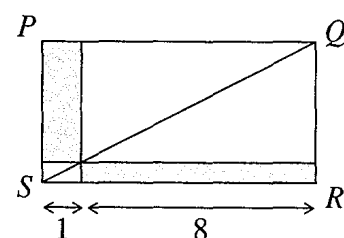
A 60                      B 75                      C 90                      D 120                      E more information needed

17. I walk to the bike shop at 3 miles per hour and cycle back along the same route at 12 miles per hour. What is my average speed, in miles per hour, for the time I am actually travelling on the route?

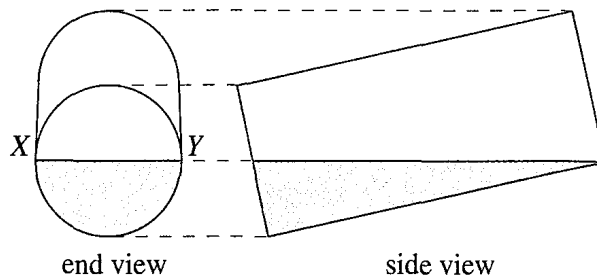
A 3.75                      B 4.8                      C 6                      D 8                      E 9

18. What fraction of the rectangle  $PQRS$  is shaded?

A  $\frac{16}{81}$                       B  $\frac{4}{9}$                       C  $\frac{2}{9}$                       D  $\frac{1}{8}$                       E  $\frac{1}{9}$

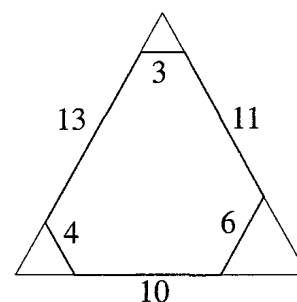


19. A cylindrical can contains lemonade, shown shaded on the diagram in which  $XY$  is a diameter. What fraction of the volume of the can is filled with lemonade?
- A just below a quarter  
 B just above a quarter  
 C exactly a quarter  
 D just below a half  
 E exactly a half



20. Which of the following scores is it impossible to achieve in this challenge?  
 (Note that if two or more answers are given to a question then they are treated as one incorrect answer.)
- A 126            B 127            C 128            D 129            E 130
21. This year started on a Tuesday. In which of the following years will each date fall on the same day of the week as it falls this year?
- A 2008            B 2009            C 2012            D 2013            E 2014

22. The diagram shows an irregular hexagon with interior angles all equal to  $120^\circ$  made by cutting the corners off a piece of card in the shape of an equilateral triangle with sides of length 20 units. An identical hexagon could also be made by cutting the corners off a different equilateral triangle: what is the side length of this triangle?
- A 23    B 25    C 27    D 29    E 31



23. A heptagon is a seven-sided polygon. What is the greatest number of the following properties that a single heptagon can possibly possess?
- |  |                                      |
|--|--------------------------------------|
| Its interior angles add up to 900 degrees. | All its sides are equal.             |
| It has exactly four acute interior angles. | It has exactly one line of symmetry. |
| It has no obtuse interior angles.          |                                      |
- A 1            B 2            C 3            D 4            E 5
24. A digital clock uses two digits to display hours, two digits to display minutes and two digits to display seconds, e.g. 10:23:42. How many times between 10:00:00 and 11:00:00 on the same morning are all six digits different?
- A 120            B 240            C 360            D 480            E 600
25. Given that  $x = \frac{111110}{111111}$ ,  $y = \frac{222221}{222223}$ ,  $z = \frac{333331}{333334}$ , which of the following statements is correct?
- A  $x < y < z$     B  $x < z < y$     C  $y < z < x$     D  $z < x < y$     E  $y < x < z$